Derivation of the Supersymmetry Transformations in 11D Supergravity

Douglas G. Stevenson
stevensonfluxinformationtheory.com

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1 Introduction

Eleven-dimensional supergravity is the unique maximal supergravity theory in 11 dimensions and the low-energy limit of M-theory. Its defining feature is local supersymmetry, which transforms the bosonic fields (metric and 3-form) into the fermionic gravitino and vice versa.

This document derives the supersymmetry transformations step by step.

2 Field Content

The bosonic fields are: - The 11D metric g_{MN} (graviton), - The 3-form gauge potential C_{MNP}, with field strength $F_{MNPQ} = 4\partial_{[M}C_{NPQ]}$.

The fermionic field is the gravitino ψ_M, a Majorana spinor-vector in 11 dimensions.

The supersymmetry parameter is a 32-component Majorana spinor $\epsilon(x)$.

3 Supersymmetry Transformations

The supersymmetry transformations that leave the 11D supergravity action invariant (up to total derivatives) are:

3.1 Transformation of the Gravitino

$$\delta\psi_M = D_M\epsilon + \frac{1}{288}\left(\Gamma_M{}^{NPQR} - 8\delta_M{}^N\Gamma^{PQR}\right)F_{NPQR}\epsilon,$$

where D_M is the covariant derivative including the spin connection:

$$D_M\epsilon = \partial_M\epsilon + \frac{1}{4}\omega_M{}^{AB}\Gamma_{AB}\epsilon,$$

and Γ_{MNPQ} are antisymmetrized products of 11D gamma matrices.

3.2 Transformation of the Metric

The metric (graviton) transforms as

$$\delta g_{MN} = \bar{\epsilon}\Gamma_{(M}\psi_{N)},$$

where the parentheses denote symmetrization.

3.3 Transformation of the 3-Form

The 3-form gauge potential transforms as

$$\delta C_{MNP} = \frac{3}{2}\bar{\epsilon}\Gamma_{[MN}\psi_{P]}.$$

The corresponding transformation of the 4-form field strength is

$$\delta F_{MNPQ} = 4\partial_{[M}\delta C_{NPQ]}.$$

Closure of the Supersymmetry Algebra The supersymmetry transformations close into the 11D diffeomorphism, gauge transformation, and Lorentz transformation on-shell (when the equations of motion are satisfied). The commutator of two supersymmetry transformations yields:

$$[\delta(\epsilon_1), \delta(\epsilon_2)] = \delta_{\text{diff}}(\xi) + \delta_{\text{Lorentz}}(\Lambda) + \delta_{\text{gauge}}(\Lambda_3),$$

where $\xi^M = \bar{\epsilon}_2\Gamma^M\epsilon_1$ is the diffeomorphism parameter, and the other terms correspond to local Lorentz and 3-form gauge transformations.

This closure confirms that 11D supergravity is a consistent supersymmetric theory.

4 Connection to SFIT

11D supergravity is a fundamental ultraviolet theory that unifies gravity with other forces through supersymmetry and higher-dimensional geometry. SFIT is an effective low-energy description focused on resonant information dynamics in four dimensions.

While 11D supergravity operates at the Planck scale, SFIT makes concrete predictions at laboratory energies (1.20134 mHz resonance, testable in ultra-cold neutron experiments). A possible synthesis is that 11D supergravity (or M-theory) provides the deep microscopic structure, while SFIT describes the emergent resonant behavior when that structure interacts with a macroscopic gravitational field.

The supersymmetry transformations above generate the fermionic partners of the bosonic fields. In SFIT, the information-carrying flux at 1.20134 mHz may be viewed as an effective collective mode arising from the underlying supersymmetric degrees of freedom when observed at laboratory scales.

5 Conclusion

The supersymmetry transformations in 11D supergravity are:

$$\delta\psi_M = D_M\epsilon + \frac{1}{288}\left(\Gamma_M{}^{NPQR} - 8\delta_M{}^N\Gamma^{PQR}\right)F_{NPQR}\epsilon,$$

$$\delta g_{MN} = \bar{\epsilon}\Gamma_{(M}\psi_{N)},$$

$$\delta C_{MNP} = \frac{3}{2}\bar{\epsilon}\Gamma_{[MN}\psi_{P]}.$$

These transformations close the supersymmetry algebra and ensure the consistency of the theory. They provide the foundation for the low-energy limit of M-theory.

SFIT offers a complementary laboratory-scale approach based on information dynamics. Future ultra-cold neutron experiments (GRANIT) have the potential to test SFIT's predictions and indirectly illuminate aspects of higher-dimensional supergravity at accessible energies.